Quantitative Finance

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Introduction and Simple Interest Rate

ISEG 100 1911-2011 ANOS

Course aims

- To develop the student's understanding of basic concepts and terminology of financial mathematics;
- To enhance the students ability to solve practical problems; and
- To understand the financial mathematical concepts necessary for other courses dealing with finance, insurance and investments.

Program

1. Simple interest
1.1 Types of time and interest
1.2 Future value at simple interest
1.3 Present value at simple interest
1.4 Simple interest debt instruments
1.5 Equation of value
1.6 Equivalent time

2. Discount interest
2.1 Comparing simple and discount interest
2.2 Discount applications – Treasury Bills

3, Compound Interest
3.1 Compound interest – Future Value Formula
3.2 Nominal rates and effective interest
3.3 Finding the Compound rate
3.4 Finding the time for an investment to grow
3.5 Equations of Value to Find the unknown
3.6 Continuous compounding

4. Ordinary Annuities
4.1 The future value of an ordinary annuity
4.2 The Present Value of an Ordinary Annuity
4.3 The Periodic Payment or Rent for an Ordinary Annuity

5. Other Annuities Certain
5.1 Deferred Annuities
5.2 Perpetuities;

6. Variable Payment Annuities
6.1 Arithmetic
6.2 Geometric

7. Amortisation of Debts and Amortisation Schedules

8. Investing in bonds

9. Leasing

10. Shares valuation

Bibliography

Gary. G. & Larry D. (2009), Mathematics of Interest Rates and Finance, Pearson, London (Recommended);

Broverman, S.A. (2008). Mathematics of Investment and Credit, ACTEX Academic Series, ACTEX Publications Inc., Winsted, Connecticut, USA

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Principal, Time and Interest

The **available income** of people may be applied in two main forms:

Consumption: total expenditure in goods or services that have a defined time of life, which does not permit any return on that which has been spent.

Saving: that may reinvested in **liquid currency** without any type of income or by **investment** (application in the form of real property or financial assets with the intention of attaining an income).

Principal, Time and Interest

Capitalization is the transformation, provoked by **time**, of the **capital** into **capital** and **interest**.

Principal, amount of money invested or borrowed (present value). The principal is a *stock* variable and <u>is always related to a moment</u>, at the beginning or the end of the period of capitalization;

Interest, cost or charge for the use of borrowed money. The Interest is a flow variable, is always related to the period of capitalization. The amount of interest charged for a loan is usually expressed as a percent of the original principal for a given time period.

The **time** (**term**), length of the loan in time units (period) corresponding to the rate. The term refers to the stated period during which the capital is applied and will be analyzed on a periodic base. The period will be annual, semi-annual, quarterly, etc., correspondingly the unit of time considered is respectively, the year, the semester, the quarter, etc.

Principal, Time and Interest

The period will be annual, semi-annual, quarterly, etc., correspondingly the unit of time considered is respectively, the year, half a year, quarter of the year, etc.

The the p**ayment period could be:**

- Annually once a year
- Semi-annually Twice a year
- Quarterly 4 times a year
- Monthly 12 times a year
- …

Principal, Time and Interest

1st Golden rule of financial mathematics

The presence of principal and the presence of time and the absence of interest is an impossibility in financial mathematics. The absence of principal or the absence of time and the presence of interest is another impossibility.

2nd Golden rule of financial mathematics

Any mathematical operation on two or more capitals requires homogenization in time. That is capitals cannot be added unless they are valued at the same point in time.

3rd Golden rule of financial mathematics

The interest in each period of capitalization is equal to the principal at the beginning of the period multiplied by the interest rate.

Time and Interest

Approximate Time: each month is assumed to be 30 days with exact time used for any portion of a month ;

Ordinary Interest: the length of a year is assumed to be 360 days (Bankers Rule);

Exact Time: every day of the term except the first day**;**

Exact Interest: the length of a year is taken as 365 days (366 for a leap year, February = 29 days)

Simple Interest

Capitalization in regime of simple interest - the interest produced in each period is excluded from the capitalization process. Within this regime, two sub-regimes may be distinguished – **the regime of simple interest** (the interest produced in each period is paid) and **the regime of interest** (the interest produced in each period is retained).

Thus interest gains are constant throughout the period of time, <u>the interest of each period only varies if the interest rate varies.</u>

Simple Interest

The interest produced throughout the period of time is given as:

where: I= interest ; P= principal; t= number of periods; i= interest rate

$$\boldsymbol{I = P \times i \times t}$$

The Maturity Value or Future Value (FV) is given as:

$$\boldsymbol{FV = P + I = P(1 + it)}$$

Simple Interest

The **Present Value at Simple Interest** (1 period):

$$P = \frac{FV}{1+it} = FV(1+it)^{-1}$$

Equation of Value:

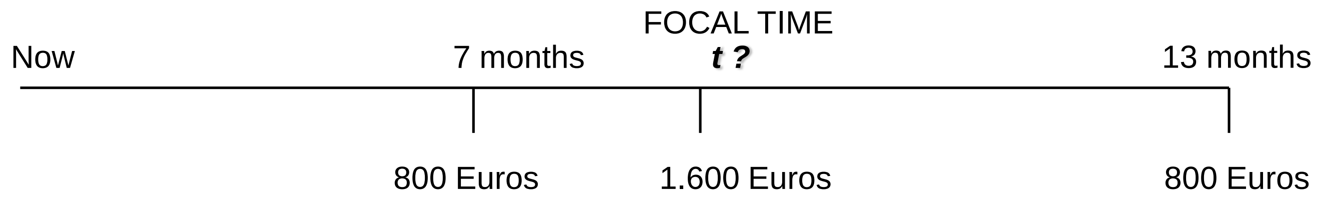
It is a mathematical expression that equates several pieces of money at some chosen date, called the focal date.

Simple Interest

Equation of Value: is a mathematical expression that equates several pieces of money at same chosen date, called focal date.

Equivalent Time: When a single payment equal the sum of the original debts, the unknown date of that payment will be called the average due date, that is usually measured in days from the present.

Example: Assume that two payments of 800 Euros are due in 7 and 13 months respectively. Using an interest rate of 8,0% find the date when a single payment of 1600 Euros.



What is the equivalent time **t** ?